TASHKENT UNIVERSITY OF TECHNOLOGY

Physics

Variant 5

CONSTRUCTOR KNOWLEDGE

Part 1

Physics. Questions 1–10

Write your answer on Answer Sheet No. 1 to the right of the corresponding question number. Where applicable, include the units of measurement. If you need to correct an answer, cross out the incorrect one, enter the question number in the table below, and write the correct answer in the field to the right.

1. Walking around Tashkent, Alex walked 3 km north, then 8 km east, and then another 3 km north. What distance did Alex move from the starting point?
2. The Titan spacecraft with a mass of 305 kg descends at a constant speed in the atmosphere of Titan. The gravitational acceleration on Titan is $1.4\,\mathrm{m/s^2}$. Find the air resistance force acting on the spacecraft.
3. During the laboratory experiment, the pressure of neon increased two times, while the concentration of gas molecules dropped 6 times. By how many times did the absolute temperature of neon increase as a result?
4. 21 calories of heat heated 0.2 g of water from zero degrees Celsius to the boiling point. Use these data to calculate how many joules are in 1 calorie. The specific heat capacity of water is $4200\,\mathrm{J/(kg\cdot{}^\circ C)}$.
5. A point charge $q = +4.0\,\mathrm{nC}$ is placed in a uniform horizontal electric field $E = 30\,\mathrm{kV/m}$. Find the magnitude of the total electric field at a point $h = 3.0\,\mathrm{cm}$ directly above the charge.
6. The voltage across a lamp depends on the current through it as $V = \alpha I^2$, where $\alpha = 3.0\,\mathrm{V/A^2}$. Find the power dissipated by the lamp for $I = 2.0\,\mathrm{A}$.
7. A simple pendulum oscillates with a period of 4 s. A small adhesive weight of the same mass is carefully stuck to the bottom of its bob in such a way that the oscillations are not disturbed. Find the new frequency of the pendulum.
8. Arthur found that the sine of the critical angle for total internal reflection at the mica–air interface turned out to be 0.625. Use these data to find the refractive index of mica. Take the refractive index of air to be 1. Give your answer as a decimal fraction.
9. The ionization energy of a hydrogen atom is 13,6 eV. Determine the wavelength of the photon that can ionize the atom.
10. A radioactive nucleus undergoes β^--decay. How do its charge number and mass number change?
    Record two digits in the answer sheet: first for charge, second for mass number.
    Use: 1 = *increases*, 2 = *decreases*, 3 = *stays the same.*

Part 2

Physics. Questions 11–14

Use Answer Sheets No. 2 to write your solutions and answers to questions 11–14. Write your solution as completely and in as much detail as possible. Remember that even if you are unable to solve the problem completely, you may still receive some points for significant progress according to the marking criteria.

11. (2 points) A 1 kg body moves along a horizontal surface under the action of a 10 N force. This force is directed upward at an angle of 30° to the horizontal. The coefficient of friction between the body and the surface is 0.2. Find the magnitude of the normal force acting on the body.

12. (3 points) Two moles of helium undergo several processes. First, the gas is heated isobarically from 22°C to 27°C, then isochorically from 27°C to 37°C, and after that isobarically again from 37°C to 42°C.

a) By how many joules did the total internal energy of the helium increase across all three processes?

b) How much heat did the helium absorb during the isochoric heating?

c) How many joules of heat did the helium absorb during the second isobaric heating?

Take the universal gas constant to be $8.3\,\mathrm{J/(mol \cdot K)}$.

13. (2 points) An electric field in a DC circuit moves a charge $q = 5\,\mathrm{C}$ through the cross-section of a heating element in time $t = 2\,\mathrm{s}$. In doing so, the external electric field does work $A = 200\,\mathrm{J}$. Determine the electric power dissipated on this heating element.

14. (3 points) A beam of light consisting of photons with momentum $p = 1{,}32 \cdot 10^{-27}\,\mathrm{kg \cdot m/s}$ is incident normally on a diffraction grating with grating spacing $d = 2\,\mu\mathrm{m}$. At what angle to the direction of the incident beam is the second order diffraction maximum observed?

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Part 1

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1. Walking around Tashkent, Alex walked 3 km north, then 8 km east, and then another 3 km north. What distance did Alex move from the starting point?

.... ***Answer:*** 10 km.

SOLUTION / PROOF :

Starting point: $(0,0)$. After 3 km north: $(0,3)$. After 8 km east: $(8,3)$. After another 3 km north: $(8,6)$. The distance from the start is $\sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100} = 10$ km.

2. The Titan spacecraft with a mass of 305 kg descends at a constant speed in the atmosphere of Titan. The gravitational acceleration on Titan is $1.4\,\mathrm{m/s^2}$. Find the air resistance force acting on the spacecraft.

.... ***Answer:*** 427 N.

SOLUTION / PROOF :

During the uniform descent of the spacecraft, the gravitational force is balanced by the air resistance force: $F_{\text{resistance}} = mg$. Substituting the numerical values from the problem, $F_{\text{resistance}} = 305 \cdot 1.4 = 427$ N.

3. During the laboratory experiment, the pressure of neon increased two times, while the concentration of gas molecules dropped 6 times. By how many times did the absolute temperature of neon increase as a result?

.... ***Answer:*** 12.

SOLUTION / PROOF :

According to the basic equation of the kinetic theory of gases, $P = nkT$. Therefore, the absolute temperature can be expressed as $T = \frac{P}{nk}$. Let the initial pressure be P and the initial concentration be n. The new pressure is $2P$, and the new concentration is $n/6$. Then $T_{\text{new}} = \frac{2P}{(n/6)k} = 12 \cdot \frac{P}{nk} = 12T$. As a result, the absolute temperature increased 12 times.

4. 21 calories of heat heated 0.2 g of water from zero degrees Celsius to the boiling point. Use these data to calculate how many joules are in 1 calorie. The specific heat capacity of water is $4200\,\mathrm{J/(kg \cdot {}^\circ C)}$.

.... ***Answer:*** 4 J.

SOLUTION / PROOF :

The formula for the heat required to heat water is $Q = cm\Delta t$. According to the problem, the boiling point is 100°C, so the temperature change is $\Delta t = 100 - 0 = 100$°C. The mass of water in kilograms is $m = 0.2\,\mathrm{g} = 0.0002\,\mathrm{kg}$. The required heat in joules is $Q_{\text{joules}} = 4200 \cdot 0.0002 \cdot 100 = 84$ J. According to the condition, this heat is equal to 21 calories: 21 cal = 84 J. Therefore 1 cal = 84/21 = 4 J.

5. A point charge $q = +4.0$ nC is placed in a uniform horizontal electric field $E = 30$ kV/m. Find the magnitude of the total electric field at a point $h = 3.0$ cm directly above the charge.

.... ***Answer:*** $E \simeq 5.0 \times 10^4$ N/C.

SOLUTION / PROOF :

The total electric field is the vector sum of the uniform external field and the field created by the point charge. The external field is horizontal:

$$\vec{E}_0 = E_0\hat{x}.$$

At the point directly above a positive charge, the field of the charge is directed vertically upward:

$$\vec{E}_q = E_q\hat{y}.$$

Its magnitude is found from Coulomb's law:

$$E_q = k\frac{|q|}{h^2}.$$

Substituting the values,

$$E_q = 9.0 \times 10^9 \frac{4.0 \times 10^{-9}}{(3.0 \times 10^{-2})^2} = 4.0 \times 10^4\ \mathrm{N/C}.$$

Therefore the total field has perpendicular components

$$E_x = 3.0 \times 10^4\ \mathrm{N/C}, \qquad E_y = 4.0 \times 10^4\ \mathrm{N/C}.$$

The magnitude is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(3.0 \times 10^4)^2 + (4.0 \times 10^4)^2} = 5.0 \times 10^4\ \mathrm{N/C}.$$

6. The voltage across a lamp depends on the current through it as $V = \alpha I^2$, where $\alpha = 3.0\,\mathrm{V/A^2}$. Find the power dissipated by the lamp for $I = 2.0$ A.

.... ***Answer:*** $P = 24$ W.

SOLUTION / PROOF :

The power dissipated by an electrical element is

$$P = IV.$$

The voltage is not given directly, but it is related to the current by

$$V = \alpha I^2.$$

Therefore

$$P = I(\alpha I^2) = \alpha I^3.$$

Substituting the values, we obtain

$$P = 3.0 \cdot (2.0)^3 = 24\ \mathrm{W}.$$

7. A simple pendulum oscillates with a period of 4 s. A small adhesive weight of the same mass is carefully stuck to the bottom of its bob in such a way that the oscillations are not disturbed. Find the new frequency of the pendulum.

.... ***Answer:*** 250 mHz.

SOLUTION / PROOF :

The formula for the period of oscillation of a simple pendulum is $T = 2\pi\sqrt{l/g}$. Hence, the frequency is $f = 1/T$. The frequency of oscillation of a simple pendulum depends only on the acceleration due to gravity g and the length of the string l. The mass of the load is not included in the formula, so adding a sticky load of the same mass does not change the frequency. The new frequency remains $f = 1/4 = 0.25$ Hz. Converting to millihertz: 0.25 Hz = 0.25 · 1000 = 250 mHz.

8. Arthur found that the sine of the critical angle for total internal reflection at the mica–air interface turned out to be 0.625. Use these data to find the refractive index of mica. Take the refractive index of air to be 1. Give your answer as a decimal fraction.

.... ***Answer:*** 1.6.

SOLUTION / PROOF :

The angle of incidence α of a light beam is measured from the normal to the interface between the media. During total internal reflection at the mica–air interface, the angle of refraction is 90°. According to Snell's law, $n_1 \sin(\alpha_{\text{crit}}) = n_2 \sin(90°)$. Substituting the values given in the problem, with $n_2 = 1$ for air, $n_{\text{mica}} \cdot 0.625 = 1 \cdot 1$. Therefore $n_{\text{mica}} = 1/0.625 = 1/(5/8) = 8/5 = 1.6$.

9. The ionization energy of a hydrogen atom is 13,6 eV. Determine the wavelength of the photon that can ionize the atom.

.... ***Answer:*** 91.2 nm.

SOLUTION / PROOF :

The energy of a photon is related to its wavelength by

$$E = \frac{hc}{\lambda}.$$

Therefore

$$\lambda = \frac{hc}{E}.$$

Taking into account the constant $hc = 1240$ eV · nm, substitute the numerical values: $\lambda = 1240/13{,}6 \approx 91{,}2 \approx 91$ nm.

10. A radioactive nucleus undergoes β^--decay. How do its charge number and mass number change?
Record two digits in the answer sheet: first for charge, second for mass number.
Use: 1 = *increases*, 2 = *decreases*, 3 = *stays the same.*

.... ***Answer:*** *13*

SOLUTION / PROOF :

In β^--decay, a neutron inside the nucleus transforms into a proton, an electron, and an electron antineutrino:

$$n \to p + e^- + \bar{\nu}.$$

The nuclear charge, or atomic number Z, increases by 1 because a proton is created, so this corresponds to digit 1. The mass number A remains unchanged because the total number of nucleons does not change; a neutron is replaced by a proton. This corresponds to digit 3. Therefore the answer is 13.

Physics. Questions 11–14

Use Answer Sheets No. 2 to write your solutions and answers to questions 11–14. Write your solution as completely and in as much detail as possible. Remember that even if you are unable to solve the problem completely, you may still receive some points for significant progress according to the marking criteria.

11. (2 points) A 1 kg body moves along a horizontal surface under the action of a 10 N force. This force is directed upward at an angle of 30° to the horizontal. The coefficient of friction between the body and the surface is 0.2. Find the magnitude of the normal force acting on the body.

.... ***Answer:*** 5 N.

SOLUTION / PROOF :

Let us analyze the forces acting on the body: the gravitational force mg is directed vertically downward, the normal force N is directed vertically upward, the external force F acts at an angle $\alpha = 30°$ upward to the horizontal, and the friction force acts opposite to the direction of motion. Newton's second law in projection onto the vertical axis gives

$$N + F\sin\alpha - mg = 0 \quad \Rightarrow \quad N = mg - F\sin\alpha.$$

Substituting the numerical values from the problem,

$$N = 1 \cdot 10 - 10 \cdot \sin(30°) = 10 - 10 \cdot 0.5 = 5\ \mathrm{N}.$$

Since $N > 0$, the body does not lift off the surface.
Marking Criteria: 2 points – the detailed solution is completely correct, and the right answer is obtained. 1 point – the applicant correctly found $N = mg - F\sin\alpha$, but made a calculation error when determining the final value. 0 points – the applicant incorrectly assumed that the body moves at a constant speed and equated $F\cos\alpha$ directly to the friction force, or the solution does not meet the criteria, or the physical laws used are incorrect.

12. (3 points) Two moles of helium undergo several processes. First, the gas is heated isobarically from 22°C to 27°C, then isochorically from 27°C to 37°C, and after that isobarically again from 37°C to 42°C.

a) By how many joules did the total internal energy of the helium increase across all three processes?

b) How much heat did the helium absorb during the isochoric heating?

c) How many joules of heat did the helium absorb during the second isobaric heating?

Take the universal gas constant to be $8.3\,\mathrm{J/(mol \cdot K)}$.

.... ***Answer:*** *1)* 498 J; *2)* 249 J; *3)* 207.5 J.

SOLUTION / PROOF :

a) Across all three processes, the total temperature change is $\Delta t = 42 - 22 = 20$°C. The internal energy of monatomic helium depends only on the initial and final temperatures:

$$\Delta U = 1.5 \cdot \nu R\Delta t = 1.5 \cdot 2 \cdot 8.3 \cdot 20 = 498\ \mathrm{J}.$$

b) During isochoric heating, the gas performs no work; all heat goes into changing the internal energy. The temperature change in this step is $\Delta t = 37 - 27 = 10$°C. Thus

$$Q_{\text{isochoric}} = 1.5 \cdot \nu R\Delta t = 1.5 \cdot 2 \cdot 8.3 \cdot 10 = 249\ \mathrm{J}.$$

c) The amount of heat during isobaric heating of a monatomic gas is $Q_{\text{isobaric}} = 2.5 \cdot \nu R\Delta t$. The temperature change in the second isobaric step is $\Delta t = 42 - 37 = 5$°C. Hence

$$Q_{\text{isobaric}} = 2.5 \cdot 2 \cdot 8.3 \cdot 5 = 207.5\ \mathrm{J}.$$

Marking Criteria: 3 points – steps 1, 2, and 3 are solved correctly. 2 points – steps 1, 2, and 3 are solved correctly in general algebraic form, but there are calculation errors, or any 2 out of 3 steps are solved correctly. 1 point – any 1 out of 3 steps is solved correctly, or the fundamental formulas for thermodynamic processes are written correctly, but the subsequent solution is missing or incorrect. 0 points – the solution does not meet the criteria, or the physical laws used are incorrect.

13. (2 points) An electric field in a DC circuit moves a charge $q = 5$ C through the cross-section of a heating element in time $t = 2$ s. In doing so, the external electric field does work $A = 200$ J. Determine the electric power dissipated on this heating element.

.... ***Answer:*** 100 W.

SOLUTION / PROOF :

The electric voltage across the heating element is $U = \frac{A}{q} = \frac{200}{5} = 40$ V. The value of the direct current in the circuit is $I = \frac{q}{t} = \frac{5}{2} = 2{,}5$ A. The power dissipated on the heater is $P = U \cdot I = 40 \cdot 2{,}5 = 100$ W.

14. (3 points) A beam of light consisting of photons with momentum $p = 1{,}32 \cdot 10^{-27}\,\mathrm{kg \cdot m/s}$ is incident normally on a diffraction grating with grating spacing $d = 2\,\mu$m. At what angle to the direction of the incident beam is the second order diffraction maximum observed?

.... ***Answer:*** $\varphi \approx 30.1°$.

SOLUTION / PROOF :

For normal incidence, the diffraction grating equation is

$$d\sin\varphi = m\lambda.$$

The photon momentum–wavelength relation is

$$p = \frac{h}{\lambda} \quad \Rightarrow \quad \lambda = \frac{h}{p}.$$

Substituting λ into the grating equation gives

$$d\sin\varphi = m \cdot \frac{h}{p}, \qquad \sin\varphi = \frac{mh}{dp}.$$

Substitution of numerical values gives

$$\sin\varphi = \frac{2 \cdot 6{,}63 \cdot 10^{-34}}{2 \cdot 10^{-6} \cdot 1{,}32 \cdot 10^{-27}} \approx 0{,}5.$$

Therefore

$$\varphi = \arcsin(0.5023) \approx 30.1°.$$